# TDA Reading Group HT 2019

## January 25

Oliver Vipond

### A Motivating Problem and Sheaf Theory Preliminaries

We will introduce an example data analysis problem which the machinery of sheaf theory is well adapted to approach. This will be followed by an introduction to sheaf theory with plentiful examples including sheafification, pushforward, pullback.

References: Applications of Sheaf Cohomology and Exact Sequences on Network Codings, Sheaves Cosheaves and Applications

## February 1

Oliver Vipond

### Network Coding Sheaves

We shall use the theory introduced in the first session to give a sheaf theoretic formulation of a network coding problem. We will then look at sheaf cohomology and how this relates to information flows on our network. Finally we'll see how Excision, Mayer-Vietoris and relative cohomology correspond to properties of our network.

References: Applications of Sheaf Cohomology and Exact Sequences on Network Codings

## February 8

Oliver Vipond

### Cosheaves, Reeb Graphs and Interleavings

This week we will meet cosheaves the dual notion to sheaves. We will revisit the Reeb Graph from last term, a graph one can associate to a filtered topological space, which has seen successful applications to analysis of big data sets. We shall see that one can view the Reeb Graph of a space as a cosheaf over the real line, which facilitates defining an algebraic distance on Reeb Graphs, the interleaving distance. We will explore how the algebraic notion of interleaving corresponds to a geometric smoothing operation on the Reeb graphs.

References: Categorified Reeb Graphs

## February 15

Nicholas Sale

### Sheaves are the canonical data structure for sensor integration

Following Michael Robinson's paper "Sheaves are the canonical data structure for sensor integration", I will present a series of "axioms" for sensor systems which make it increasingly clear that what we want to describe is a sheaf. I will then state the data fusion problem in the sheaf-theoretic context, before talking about the contributions Robinson makes towards defining "approximate" solutions to data fusion, and interpreting non-trivial sheaf cohomology as obstructions to globally consistent fusions.

References: Sheaves are the canonical data structure for sensor integration, Tutorial on Sheaves in Data Analytics

## March 1

Oliver Vipond

### Toward a Spectral Theory of Cellular Sheaves

Motivated by the far reaching applications of spectral graph theory, we shall explore higher dimensional Laplacian operators analogous to the graph Laplacian. Higher dimensional combinatorial Laplacian operators and their spectra are an area of active research, and provide an insight into the algebraic-topological features of higher dimensional complexes. We shall explore the spectra of Laplacian operators on cellular sheaves and their relation to sheaf cohomology. We will see similarities and disparities between properties of the graph Laplacian and the higher dimensional counterparts. In particular we will see that the spectra are preserved under sufficiently nice pullbacks and push forwards. We will conclude with a summary of potential applications and open questions in spectral sheaf theory.

Notes: Toward a Spectral Theory of Cellular Sheaves

References: Toward a Spectral Theory of Cellular Sheaves

## March 8

Oliver Vipond

### Persistent Local Systems

This week we will look at recent work from Robert MacPherson and Amit Patel which generalises ideas of persistent homology from real-valued filter functions to manifold-valued maps. Using new sheaf theoretic methods, they study the homology of the fibres of such manifold valued maps and show that one can produce non-trivial stable lower bounds for the homology of these fibres. The analogue of the barcode from the real-valued setting is a locally constant sheaf which is termed the persistent local system. We shall show how to construct a persistent local system given a manifold-valued map and see an example of such a map with a non-trivial persistent local system.

Notes: Persistent Local Systems

Reference: Persistent Local Systems