Morse Theory - Trinity Term 2019

May 2019

The syllabus sketched out below is a guideline for the topics I plan to cover during the 8 weeks of this term. There is scope to change topics as informed by your interests as a group. I have provided some references for each of the topics - this is by no means an exhaustive list - although neither do I expect you to find time to read every reference cover to cover!

The course will require independent study, however I encourage you to ask each other and myself for help and support if you encounter difficulties. Whilst you are welcome to work on problem sets together you should write up solutions independently.

If the pacing of the course seems too slow or too fast you should let me know so we can adapt the content appropriately.

1 Syllabus

1. Course prerequisites
   (a) Differential Geometry [Lee03, Hit]
   (b) Algebraic Topology [Hat02]
   (c) Functional Analysis [Sei]
2. Introduction to Morse Theory [Mil60, Nic, AD14]
   (a) Critical points and homotopy type
   (b) Morse Lemma
   (c) Morse Inequalities
3. Morse Homology [Mil60, Nic, AD14]
   (a) The Morse Complex
4. Morse Homology [Mil60, Nic, AD14]
   (a) Poincare Duality
   (b) Kunneth Formula
5. Lefchetz Hyperplane Theorem [Mil60, Nic, AD14]
6. Discrete Morse Theory [For]

(a) Discrete vector fields and collapses

(b) Combinatorial Morse Complex

7. Intro to Floer Homology [AD14]

8. Combinatorial Realisation of the Thom-Smale Complex via Discrete Morse Theory [Gal10]

References

[AD14] Michle Audin and Mihai Damian. *Morse Theory and Floer Homology*. Universitext, Virtual Series on Symplectic Geometry. Springer-Verlag, London, 2014.

[For] Robin Forman. A USERS GUIDE TO DISCRETE MORSE THEORY.

[Gal10] tienne Gallais. Combinatorial realization of the Thom-Smale complex via discrete Morse theory. *Annali della Scuola Normale Superiore di Pisa - Classe di Scienze*, 9(2):229–252, 2010.

[Hat02] A. Hatcher. *Algebraic Topology*. Algebraic Topology. Cambridge University Press, 2002.

[Hit] Nigel Hitchin. DIFFERENTIABLE MANIFOLDS.

[Lee03] John M. Lee. *Introduction to Smooth Manifolds*. Graduate Texts in Mathematics. Springer-Verlag, New York, 2003.

[Mil60] John Willard Milnor. *Morse Theory: Lectures, Fall 1960*. 1960. Google-Books-ID: srsrAAAAYAAJ.

[Nic] Liviu I Nicolaescu. An Invitation to Morse Theory. page 291.

[Sei] David Seifert. C4.1 Further Functional Analysis.